of oval tube profile; x, r, φ, coordinates; ξ, drag coefficient; q_V, density of volumetric heat release; T, temperature; ρ, density; c_p, specific heat capacity; p, pressure; R, gas constant; σ, root-mean-square deviation; F, Fisher criterion; R^2, multiple correlation coefficient; Re, Reynolds number; m, porosity of the bundle relative to the heat-transfer agent; G, mass flow rate of the heat-transfer agent; q_s, heat flow density. Subscripts: st, stabilized; i, initial; h, housing; in, inlet.

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RADIANT—CONVECTIVE HEAT EXCHANGE IN TURBULENT MOTION OF A GAS SUSPENSION WITHIN A TUBE

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UDC 536.242.001.5

A calculation of the temperature field and radiant and convective components of the thermal flux density is performed for combined action of convection and radiation in a dusty gaseous medium.

At present there are available a large number of studies of the process of radiant—convective heat exchange [1-4]. However, for the case of flow of a gas suspension in a round tube this problem has been considered only in [5, 6]. Many questions such as the effect on heat exchange of the direction of the thermal flux, the parameters of the carrier gas and particles, and temperature conditions require further study.

In the general case radiant—convective heat exchange is described by a system of equations in which the energy equation is an integral-differential one. Numerical solution of the problem is possible only with significant expenditures of machine time, so that development of simple but reliable methods for engineering calculations of the radiant component of the thermal flux density on the tube surface during motion of a dusty gas therein is a problem of practical value.

The present study will present a simplified method and results of calculating radiant—convective heat exchange for flow of a gas suspension in a circular tube.

Relying on [7], we will assume that the solid particles found in the gas suspension flow are uniformly distributed over the tube section. The gas suspension is considered as a quasihomogeneous absorbing and radiating grey medium. Temperature difference between gas and particles will be neglected, as well as the effect of these temperatures on convective heat exchange. The latter assumption is satisfied well for tubes of small diameter if the particle mass flow concentration does not exceed the value two [8]. We will consider the flow of the gas suspension in a region far removed from the tube entrance. The tube wall is absolutely black. On the wall the boundary condition is q_w = const. Following [7], we write the energy equation for the gas suspension in the form

$$\rho_* w_x \frac{\partial h_*}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}(rq) + \eta_{\rm res}. \tag{1}$$

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From Eq. (1) we can obtain the following equations for calculating the temperature field and the total thermal flux density:

$$T_c - T = \frac{2q_c r_0}{\lambda_c} \int_R^1 \frac{\int_0^R \left(\frac{\rho_* w_x}{\overline{\rho_* w_x}} - \frac{r_0}{2q_c} \eta_{res} \right) R dR}{\frac{\lambda}{\lambda_c} \left(1 + \frac{\Pr}{\Pr_T} \frac{\varepsilon_\sigma}{\nu} \right) R} dR, \quad (2)$$

$$\frac{q + q_r}{q_c} = \frac{2}{R} \int_0^R \frac{\rho_* w_x}{\overline{\rho_* w_x}} R dR. \quad (3)$$

We will assume that energy transport by radiation in the axial direction is negligibly small. Then the functions $\eta_{res}(r)$ and $q_r(r)$ can be calculated with the aid of equations valid for an isothermal infinitely long cylinder filled with the given absorbing and radiating medium [9]:

$$\eta_{res}(r) = 4\alpha^2 \sigma T_c^4 r_0 \int_1^\infty \frac{K_1(\alpha r_0 y) I_0(\alpha r y)}{y} dy + 4\alpha^3 \sigma \int_0^{r_0} K(\alpha r, \alpha r_1) r_1 T^4(r_1) dr_1 - 4\alpha\sigma T^4(r), \quad (4)$$

where

$$K(\alpha r, \alpha r_1) = \begin{cases} \int_1^\infty K_0(\alpha r y) I_0(\alpha r_1 y) dy, & \text{if} \quad r_1 < r, \\ \int_1^\infty K_0(\alpha r_1 y) I_0(\alpha r y) dy, & \text{if} \quad r_1 > r, \end{cases} \quad (5)$$

$$q_r(r) = 4\alpha r_0 \sigma T_c^4 \int_1^\infty \frac{K_1(\alpha r_0 y) I_1(\alpha r y)}{y^2} dy -$$

$$- 4\alpha^2 \sigma \int_0^r r_1 T^4(r_1) dr_1 \int_1^\infty \frac{K_1(\alpha r y) I_0(\alpha r_1 y)}{y} dy + 4\alpha^2 \sigma \int_r^{r_0} T^4(r_1) r_1 dr_1 \int_1^\infty \frac{K_0(\alpha r_1 y) I_1(\alpha r y)}{y} dy.$$

With the aid of Eqs. (2)-(5) calculations of T(r), q(r), and $q_r(r)$ were performed for a flow in a tube 20 mm in diameter of a gas mixture (nitrogen, argon, or helium) with 50-μm-diameter carbon particles. The particle mass flow concentration varied from 0 to 2. The maximum wall temperature upon heating of the gas suspension was 2000 K, and the minimum wall temperature with cooling was 300 K. The gas pressure was 10^6 Pa, with thermal flux density at the wall of $1.5 \cdot 10^5 - 5 \cdot 10^5$ W/m². In the majority of calculations the Reynolds number was equal to 52,000.

In the temperature field calculations the dependence of the gas physical properties on temperature was considered. For constant properties the turbulent viscosity was calculated with the Reichardt expression, while the effect of temperature inhomogeneity on the ratio ε_σ/ν was considered as in [10]. It was assumed that the solid particles had no effect on the turbulence. The turbulent Prandtl number was taken constant and equal to 0.9.

Scattering of radiation by the solid particles was considered approximately by the method proposed in [11]. At diffraction parameter values corresponding to the calculation conditions anisotropic scattering occurs, with "forward" scattering dominating. The fraction of radiation diffracted was equal to 0.75. In this case, as in [11], the effect of scattering on the heat exchange process is less significant than with isotropic scattering, and can be considered approximately by multiplying the absorption coefficient by some other coefficient less than unity, which depends on the effective Schuster number. The dimensionless true absorption coefficient was taken equal to 0.6, and the correction coefficient to 0.75.

The temperature field was calculated by the iteration method. In the initial approximation it was assumed that $\eta_{res} = 0$ with the physical properties of the gas being constant. In each subsequent approximation radiant heat exchange and the temperature dependence of the gas physical properties were considered. Preliminary calculations revealed that much time was required to calculate temperatures with consideration of η_{res}. Therefore, a quite coarse division of the tube over radius was selected, at the nodes of which the η_{res} values were

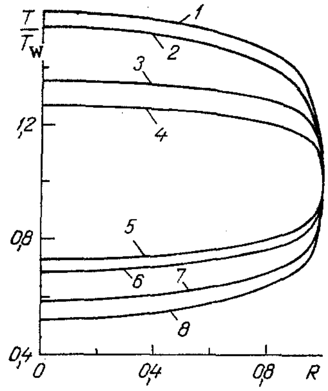

Fig. 1. Temperature distribution over tube radius for cooling (curves 1-4) and heating (curves 5-8) of the gas suspension for Re = 52,000 and $q_w = 1.5 \cdot 10^5$ W/m²: 1, 8) K = 0; 2, 7) 0.2; 3, 6) 1; 4, 5) 2.

calculated directly with Eq. (4). The results obtained were transformed to a finer grid by the spline-interpolation method. Direct calculation of η_{res} at 50 nodes with subsequent interpolation to a 200-point grid allowed reduction in calculation time compared to direct calculation of η_{res} with Eq. (4) at 200 points by an order of magnitude.

The calculations showed that radiant heat exchange in the gas suspension flow leads to temperature equalization over tube section for both heating and cooling (Fig. 1). Analysis revealed that for a given q_w value with increase in concentration the fraction of thermal flux transferred by radiation increases. The convective component of the thermal flux proves to be 3-5% higher than that calculated by the Newton–Richman law. In other words, deformation of the temperature field due to radiant heat exchange has a weak effect on the value of the convective heat liberation coefficient given the condition that Re = idem, T_w = idem, $\bar{T}_{idem}$.

In all cases the distribution of the total thermal flux density over radius was close to linear, i.e., remained the same as for a pure gas.

Figure 2 shows the distribution of the radiant thermal flux component over tube radius, as obtained by calculating q_r by two methods: with and without consideration of radiant–convective interaction. In the first case the quantity q_r was calculated with the temperature field obtained with Eq. (2) with consideration of η_{res}, while in the second case the condition $\eta_{res} = 0$ is used. As is evident from Fig. 2, calculation of radiant fluxes without consideration of radiant–convective interaction produces elevated results. While for heating the difference is relatively small, for cooling it reaches 100%. The results obtained can be explained by comparing Fig. 2 with Fig. 1. Calculation by the first method was performed for curves with K > 0 (Fig. 1) and by the second, for K = 0. The largest contribution to the resultant radiant flux is produced by those layers which have the highest temperature. While for heating with change in K the temperature of these layers changes little ($T \approx T_w$), for cooling the change is significant (the temperature of these layers is close to that of the flow core).

The fraction of the thermal flux transferred by radiation for a fixed number Re depends significantly on the physical properties of the gas, for example, K = 2, T_w = 1300 K, and Re = 52,000 for argon $q_{rw}/q_w = 0.47$ (for $\bar{T}$ = 928 K), for nitrogen, 0.25 (for $\bar{T}$ = 873 K), for helium, 0.016 (for $\bar{T}$ = 1070 K). The effect of the gas physical properties manifests itself in the convective heat liberation coefficient (which is smallest for argon) and the density, upon which the absorption coefficient depends. Since argon has the highest density, for a given value of K for the argon–carbon particle suspension the volume concentration of particles is highest, and thus, the absorption coefficient is highest.

The effect of the Buger number on the radiant thermal flux component at the wall is shown in Fig. 3. It is evident that with increase in the number Bu $q_{rw}/(\sigma T_*^4)$ increases, although with approach to the value Bu = 1 the rate of increase in radiant thermal flux decreases. Energy transport by radiation begins to appear most markedly at optical thickness (Buger number) values close to 1...2, while for Bu ≫ 1 (optically thick layer) $q_{rw}/(\sigma T_*^4) \to 0$ [12].

The q_{rw} values obtained in the calculations were compared with the expression

$$q_{rw}^0 = \sigma\varepsilon (T_w^4 - \bar{T}^4), \tag{6}$$

where $\varepsilon = 1 - \exp(-\alpha \ell_{ef})$, and $\ell_{ef} = 0.9d$.

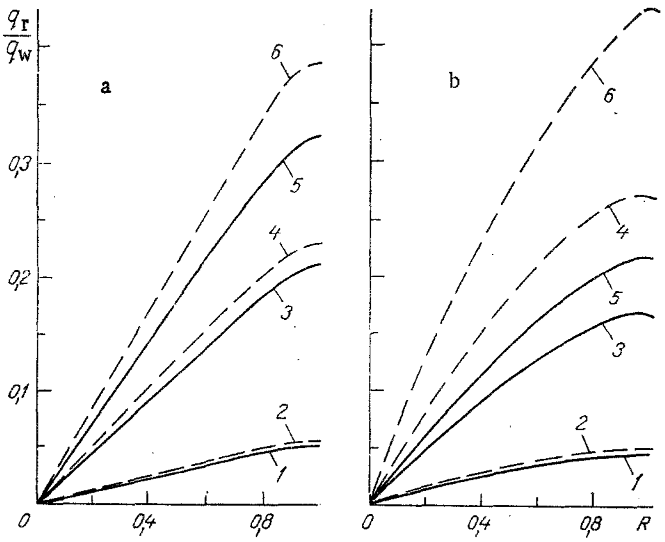

Fig. 2

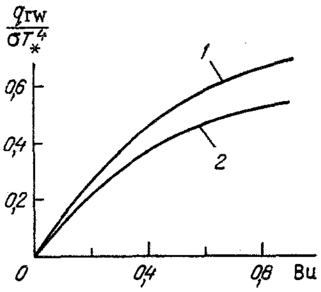

Fig. 3

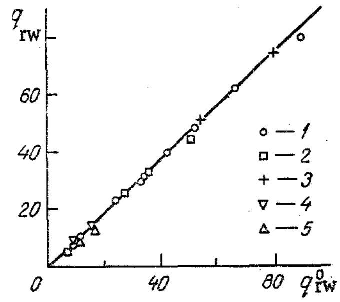

Fig. 4

Fig. 2. Resultant radiant flux density distribution over tube radius for heating (a) and cooling (b) for Re = 52,000 and $q_w = 1.5 \cdot 10^5$ W/m^2 (solid lines, with; dashed lines, without consideration of radiant–convective interaction): 1, 2) K = 0.2; 3, 4) 1; 5, 6) 2.

Fig. 3. Dimensionless resultant flux density on wall vs Buger number: 1) cooling, $T_w/\overline{T} = 0.4$ ($T_* = \overline{T}$); 2) heating, $T_w/\overline{T} = 1.5$ ($T_* = T_w$).

Fig. 4. Comparison of calculation results with Eq. (7): 1) nitrogen, heating; 2) nitrogen, cooling; 3) argon, heating; 4) helium, heating; 5) helium, cooling. q_{rw}, q_{rw}^0, kW/m^2.

Results of this comparison are shown in Fig. 4, whence it follows that the data produced by the exact calculations are described satisfactorily by the expression

$$q_{rw} = 0.91 q_{rw}^0. \tag{7}$$

Thus, the results of the numerical study of radiant–convective heat exchange performed above show that for engineering calculations of thermal fluxes on the wall of a tube within which a dusty gas flows it is acceptable to use the "one-dimensional" approximation, according to which the radiant component of the thermal flux density is calculated with Eq. (7) and the convective component is found by the expression

$$q_{cw} = \alpha_c (T_w - \overline{T}), \tag{8}$$

while the convective heat liberation coefficient α_c is found by expressions valid for purely convective heat exchange. It was assumed in the present study that the solid particles have no effect on the value of α_c. For approximate evaluation of α_c with consideration of the effect of the solid particles the expressions presented in [8] can be recommended. The effect of the temperature factor must then be considered just as in a pure gas. It can be proposed that for the case of a nonblack wall at emissivities of $\varepsilon_w = 0.8$-0.9, a correction factor equal to $0.5(\varepsilon_w + 1)$ should be introduced on the right side of Eq. (6).

NOTATION

ρ_*, h_*, mixture density and enthalpy; r, r_0, radial coordinate and tube radius; $R = r/r_0$, dimensionless radius; η_{res}, resultant radiation flux volume density; q_r, resultant radiation flux density; α, absorption coefficient; $\overline{T}$, mean mass temperature of mixture; $T_* = \overline{T}$, if $T_w < \overline{T}$; $T_* = T_w$, if $T_w > \overline{T}$; K, particle mass flow concentration; $I_0(x)$, $K_0(x)$, $K_1(x)$, $I_1(x)$, modified Bessel functions; ε, particle cloud emission coefficient; ℓ_{ef}, effective beam length; σ, Stefan–Boltzmann constant; $Bu = \alpha r_0$, Buger number. Subscript w, wall.

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UTILIZATION OF THE K–ε TURBULENCE MODEL IN A FREE-CONVECTIVE TURBULENT BOUNDARY LAYER

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A free-convective turbulent boundary layer on a vertical isothermal surface is examined. The influence of the buoyancy force on the kinetic energy of the turbulent fluctuations is analyzed. A modification is proposed for the turbulence model that takes account of the free-convective flow singularities.

Modeling turbulence when studying free-convective boundary layers is based mainly on the analogy with forced flows [1, 2] without taking account of the influence of the lift force on the turbulent characteristics. Experimental papers [3-6] that have recently appeared and in which the structure of a turbulent free-convective flow is investigated in detail permitted substantial refinement of the turbulence model and taking account of the singularities of similar flows.

As the initial equations to describe the free-convective flow around an isothermal vertical surface, the turbulent boundary-layer equations in a Boussinesq approximation were used. Details of the problem formulation can be found in [7].

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